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Optimization of Probe Excitation for a Two-Probe Measurement of the Refractive-Index Structure Function Constant of the Atmosphere

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CONTENTS

Abstract ii
Problem Status ii
Authorization ii

INTRODUCTION 1

THEORY 1

APPLICATION 5

CONCLUSIONS 6

ABSTRACT

This study considers the tradeoffs involved in selecting the excitation power for a pair of fine-wire resistance thermal probes for the measurement of the atmospheric refractive-index structure function constant C_N. It is shown that the choice is critical if valid measurements are to be made down to 10^{-8} $m^{-1/3}$. The optimum probe excitation found is approximately one microwatt, which is consistent with earlier NRL practice but inconsistent with generally accepted values used elsewhere which range upwards from 10 microwatts.

PROBLEM STATUS

A final report on one phase of a continuing NRL problem.

AUTHORIZATION

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OPTIMIZATION OF PROBE EXCITATION FOR A TWO-PROBE MEASUREMENT OF THE REFRACTIVE-INDEX STRUCTURE FUNCTION CONSTANT OF THE ATMOSPHERE

INTRODUCTION

Fine-wire resistance probes are widely used for sensing rapid fluctuations in fluid temperatures or velocities. Typical wire sizes range from 1 to 50 μm in diameter and 1 to 5 mm in length in a variety of materials, including platinum, tungsten, nickel, and some alloys. For sensing fluid velocities the probes are operated hot by exciting them with a relatively large bias current. For sensing fluid temperature, however, the self-heating of the probe introduces an unwanted velocity sensitivity which must be minimized by operating the probes with as little excitation as possible. One cannot make the excitation arbitrarily small, however, because the inherent Johnson noise of the probe produces a background noise level which increases relative to the signal with decreasing excitation.

This report examines the tradeoff and determines the optimum excitation level and the minimum observable signal for the case where two probes are operated side by side about 10 cm apart and the desired signal comprises the temperature-difference fluctuations over a given bandwidth. This case is important because it represents a practical method of measuring the refractive-index structure function constant C_N of the atmosphere. The case of two probes is different from that of one probe because the wind and temperature fluctuations of the two probes are partially correlated, whereas the Johnson-noise voltages of the two probes are completely uncorrelated. Thus in the two-probe case the Johnson noise is relatively more important, and the optimum excitation turns out to be greater than in the single-probe case by a factor in the vicinity of two.

THEORY

The measured temperature difference $\Delta\widetilde{\theta}$ can be expressed as

$$\Delta\widetilde{\theta} = \Delta\theta + C_w \Delta w + \frac{\sqrt{2}\, N V_j T_0}{I R_0}, \tag{1}$$

where $\Delta\theta$ is the actual temperature difference, C_w is the sensitivity of the probe temperature to the wind, and Δw is the difference between the wind velocities at the two probes. The factor $\sqrt{2}$ accounts for the Johnson noise being from both probes; N is a number a bit over unity to account for the noise figure of the sensing amplifiers used; V_j is the Johnson-noise voltage inherent in a single probe; R_0 is the probe resistance at temperature T_0; and I is the excitation current in the probe. N can be expressed as

$$N = 10^{N.F./20\text{dB}}, \tag{2}$$

where $N.F.$ is the noise figure of the amplifiers. The measured quantity is the variance of $\Delta\widetilde{\theta}$:

$$\langle \Delta\widetilde{\theta}^{\,2} \rangle = P_s + P_n, \tag{3}$$

where P_s is the signal power and P_n is the noise power:

$$P_s = \langle \Delta\theta^2 \rangle \tag{4a}$$

$$P_n = C_w \alpha \sqrt{\langle \Delta\theta^2 \rangle \langle \Delta w^2 \rangle} + C_w^2 \langle \Delta w^2 \rangle + \frac{2N^2 {T_0}^2}{I^2 {R_0}^2} \langle {V_j}^2 \rangle. \tag{4b}$$

Equation (4b) shows the effects of a correlation factor α between the wind fluctuations and the temperature fluctuations. This factor ranges roughly from +0.7 to −0.7, depending on the thermal vertical gradient. It is typically negative during the day and positive at night.*

To optimize the probe excitation, one assumes that a minimum required signal-to-noise ratio H is specified, and one selects the excitation to minimize the signal level at which the specified signal-to-noise ratio obtains. Thus one requires that

$$P_s = HP_n \tag{5}$$

at the smallest possible value of P_s. To find this value, one must express Eq. (5) with all excitation dependencies explicit. Starting with the wind-caused temperature fluctuations t_w:

$$t_w \equiv C_w \sqrt{\langle \Delta w^2 \rangle}, \tag{6}$$

we must find the worst case (largest) value that this quantity can assume. An empirical equation from which C_w can be obtained is†

$$\frac{P}{t_r} = \pi k \ell \left[0.24 + 0.56 \left(\frac{\rho d}{\mu} W \right)^{0.45} \right], \tag{7}$$

where P is the probe excitation power, t_r is the temperature rise of the wire, ℓ is the wire length (0.15 cm in our case), k is the thermal conductivity of the fluid ($k_{air} = 2.53 \times 10^{-4}$ W/cm °K), ρ is the fluid density ($\rho_{air} = 1.3 \times 10^{-3}$gm/cm^3), μ is the fluid viscosity ($\mu_{air} = 1.79 \times 10^{-4}$gm/cm s), d is the wire diameter, and W is the wind velocity. Solving for t_r and differentiating,

$$C_w \equiv \frac{\partial t_r}{\partial W} = \frac{-AB}{[1 + (B\langle W \rangle)^{0.45}]^2 \, (B\langle W \rangle)^{0.55}}, \tag{8}$$

*J. Wyngaard, private communication.
*S. Corrsin, *Handbuch der Physik*, Vol. 8, 1963, p. 524.

where

$$A \equiv \frac{1.88P}{\pi k \ell} \tag{9a}$$

and

$$B \equiv \frac{6.57\rho d}{\mu}\,. \tag{9b}$$

To analyze this further, one needs a few empirical facts concerning the ratios

$$M_1 \equiv \left(\frac{\langle \Delta w^2 \rangle}{\langle {w_1}^2 \rangle}\right)^{1/2} \tag{10}$$

and

$$M_2 \equiv \frac{\langle {w_1}^2 \rangle^{1/2}}{\langle w \rangle} \tag{11}$$

where w_1 is the wind fluctuation at a single probe. These two ratios are nearly independent of the atmospheric conditions. The variance of w_1 is somewhat larger than the variance of the wind difference between probes Δw due to the spatial correlation of the wind velocity. At a 10-cm probe separation, M_1 is typically about 0.7; M_2 is typically 0.2 and is independent of the average wind velocity. If one defines

$$M \equiv M_1 M_2 = \frac{\sqrt{\langle \Delta w^2 \rangle}}{\langle W \rangle}\,, \tag{12}$$

one can use Eqs. (6), (8), and (12) to get

$$t_w = \frac{-MA\chi}{(1 + \chi)^2}, \tag{13}$$

where

$$\chi \equiv (B\langle W \rangle)^{0.45}. \tag{14}$$

The worst-case value for t_w can be found by differentiating Eq. (13) and setting the result equal to zero:

$$0 = \frac{\partial t_w}{\partial \chi} = \frac{-MA}{(1 + \chi)^4}\left[(1 + \chi)^2 - 2\chi(1 + \chi)\right]\,, \tag{15}$$

the solution for which is

$$\chi = 1. \tag{16}$$

The worst-case wind speed is therefore

$$\langle W \rangle_{wc} = \frac{1}{B} = \frac{0.152\mu}{\rho d}. \tag{17}$$

For our probes the diameter is 2 μm, and the worst-case wind speed is 3.8 km/hr, so it is indeed correct to use this worst-case value for t_w. From Eqs. (13) and (16) we have

$$t_w\big|_{\text{worst case}} = \frac{-MA}{4} = -\frac{0.47MP}{\pi k \ell} . \tag{18}$$

The Johnson noise can be expressed as

$$\langle V_j^2 \rangle = \frac{4KT^2 R_0 \Delta f}{T_0} , \tag{19}$$

where K is Boltzmann's constant and Δf is the bandwidth over which the variance in Eq. (3) is measured.

Substituting Eqs. (18) and (19) into Eq. (4b), one obtains

$$P_n = -\frac{0.47MP\alpha}{\pi k \ell}\sqrt{\langle \Delta\theta^2 \rangle} + \frac{0.22M^2P^2}{\pi^2 k^2 \ell^2} + \frac{8N^2KT^3\Delta f}{P} , \tag{20}$$

where we have used the relation $P = I^2R$.

Using Eqs. (20) and (4a), one can express Eq. (5) as

$$P_s = -\alpha EHP\sqrt{P_s} + E^2HP^2 + \frac{FH}{P} , \tag{21}$$

where

$$E \equiv \frac{0.47M}{\pi k \ell} \tag{22a}$$

and

$$F \equiv 8N^2KT^3\Delta f. \tag{22b}$$

Equation (21) shows explicitly the dependence of Eq. (5) on P_s and P. The optimum probe excitation power P_{op} is given by differentiating Eq. (20) and assuming that

$$\frac{dP_s}{dP} = 0,$$

that is,

$$0 = \alpha EH\sqrt{P_s} + 2E^2HP_{op} - \frac{FH}{P_{op}^2} . \tag{23}$$

Solving for $\sqrt{P_s}$, one obtains

$$\sqrt{P_s} = \frac{2E^2 P_{op} - \frac{F}{P_{op}^2}}{\alpha E}. \tag{24}$$

Eliminating $\sqrt{P_s}$ from Eqs. (21) and (24) gives

$$0 = (4 + \alpha^2 H)D^2 - (4 + 2\alpha^2 H)D + 1, \tag{25}$$

where

$$D \equiv \frac{P_{op}^3 E^2}{F}. \tag{26}$$

Of the two solutions for D in Eq. (25), one should choose that solution which yields a positive result for $\sqrt{P_s}$ in Eq. (24).

APPLICATION

A typical set of values for our C_n measurements is

$$M_1 = 0.7$$

$$M_2 = 0.2$$

$$\ell = 0.15 \text{ cm}$$

$$N = 1.41 \ (N.F. = 3 \text{ dB})$$

$$\Delta f = 1000 \text{ Hz}$$

$$T = 290 \ ^\circ K.$$

From these one gets

$$E = 5.52 \times 10^2 \ ^\circ K/W$$

$$F = 5.39 \times 10^{-12} \ (^\circ K)^2 W.$$

Table 1 shows the results for various choices of H and α. Since α is not expected to be more than 0.7 in magnitude, the worst case is at $\alpha = -0.7$, which indicates that, depending on the preference for H, the optimum probe excitation is in the vicinity of 1.1 to 1.3 μW. Table 2 shows the minimum observable signal for various choices of H and p over the range of α.

Table 1
Optimum Excitation Power P_{op} and Minimum Observable Signal $\sqrt{P_s}$, at Selected Values of the Signal-to-Noise Ratio H and the Wind-Temperature Correlation α

H	α	P_{op} (μW)	$\sqrt{P_s}$ °$K \times 10^{-3}$
10	1	3.09	2.39
10	0.7	2.96	3.09
10	0	2.07	6.25
10	−0.7	1.10	9.69
10	1	0.91	10.76
5	1	2.97	2.17
5	0.7	2.81	2.67
5	0	2.07	4.42
5	−0.7	1.30	6.24
5	− 1	1.10	6.87

Table 2
Minimum Observable Signal in °$K \times 10^{-3}$ at a Signal-to-Noise Ratio H of 5 for Various Values of Excitation Power P and Wind-Temperature Correlation α

α	$P = 1.3$ μW	$P = 1.5$ μW	$P = 10$ μW
1	3.36	3.00	4.7
0.7	3.73	3.40	6.1
0.4	4.16	3.87	8.1
0	4.83	4.63	12.5
−0.4	5.60	5.53	19.1
−0.7	6.24	6.30	25.4
− 1	6.94	7.14	32.4

The choice of H is 5, in the work being done at NRL and the corresponding best choice for excitation is 1.3 μW.

CONCLUSIONS

Some other experimenters have chosen values of excitation as high as 60 μW for probes in measuring the atmospheric structure function constant. This report shows the danger in going even as high as 10 μW. Of course these numerical results are specific for the case of the probes used in this work; however, the only probe property of any significance is the length; the resistivity and diameter of the probe have dropped out of the picture.

It is evident that the optimum excitation level is desirable, because the minimum observable signal that corresponds is occasionally observed in the atmosphere. Therefore it is important to use a low-noise-figure amplifier for the measurement and to use the corresponding optimum bias current.

As an interesting variation, the Johnson noise can be measured accurately and subtracted out with reasonably good accuracy. If this procedure is used, the optimum excitation will be less. The wind noise cannot be subtracted out so easily since it varies with time and since measuring it requires another probe at each original probe.

If the Johnson noise is subtracted out, the optimum bias current can still be calculated using this analysis by reducing N by the square root of the ratio of the uncorrected noise variance to the estimated corrected noise variance; i.e., if the noise variance can be measured to within 10 percent and subtracted off, the noise of the resultant will be one-tenth in variance, and N would be reduced by a factor of the square root of ten.

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13. ABSTRACT

This study considers the tradeoffs involved in selecting the excitation power for a pair of fine-wire resistance thermal probes for the measurement of the atmospheric refractive-index structure function constant C_N. It is shown that the choice is critical if valid measurements are to be made down to 10^{-8} $m^{-1/3}$. The optimum probe excitation found is approximately one microwatt, which is consistent with earlier NRL practice but inconsistent with generally accepted values used elsewhere which range upwards from 10 microwatts.

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